Shear instability in the St. Lawrence Estuary, Canada: A comparison of fine-scale observations and estuarine circulation model results

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Abstract. A three-dimensional numerical model was used to predict the timing and the location of shear instabilities in the St. Lawrence Estuary. This model suggests that significant mixing occurs during flood tides in the upper estuary. This mixing is associated with a strong bottom density current made of the cold Gulf of St. Lawrence intermediate waters flowing under the St. Lawrence mixed surface waters. Guided by these results, a field experiment was undertaken in summer 1997 to verify this and to document the conditions that favor the development of instabilities. The instabilities were found as predicted and documented from acoustic imaging, current profiler, and density measurements. The instabilities first develop in the form of wavelike disturbances before they break, like Kelvin-Helmholtz instabilities. The unstable waves have wavelength of $\approx 140 - 150$ m and extend vertically between 10 and 25 m. The fine-scale observations of the semidiurnal evolution of the vertical structure of currents and density at the experimental site are compared with the numerical results. The model reproduces accurately the tidal variability of the currents but underestimates by a factor of 2 the amplitude of the density fluctuations. The general patterns of the shear squared $S^2$ and the buoyancy frequency squared $N^2$ are reasonably well reproduced by the model, but their intensities are $\approx$ 2 times smaller than the observations. This difference is attributed to the limited vertical resolution of the model at the pycnocline. However, the modeled Richardson numbers, $Ri \equiv N^2S^{-2}$, are reasonably well reproduced and appeared to be useful for the prediction of instabilities in such a complex environment.

1. Introduction

The development of shear instabilities (or Kelvin-Helmholtz instabilities) is thought to be the primary mechanism by which mixing is enhanced within the pycnocline of partially stratified estuaries [e.g., Geyer and Smith, 1987; Geyer and Farmer, 1989; Jay and Smith, 1990; Sullivan and List, 1993; Dyer, 1997]. The resulting vertical fluxes of mass determine the mean baroclinic pressure field that drives the residual haline circulation as well as the vertical distribution of nutrients and suspended particulate matter. Knowledge of the timing, location, and the magnitude of the turbulent fluxes associated with these naturally occurring events is still very limited. Until recently, our understanding relied mostly on laboratory and simplified numerical experiments, but only on fragmentary field observations (see Gregg [1987] and Thorpe [1987] for reviews). The short time and spatial scales of shear instabilities make it difficult to localize and observe their generation in the natural environment. A few authors have managed to observe directly their occurrences in the ocean, with the use of tracers [e.g., Woods, 1968] or echo sounders [e.g., Haury et al., 1979; Farmer and Smith, 1980; Farmer and Denton, 1985; Stacey and Zedel, 1986; Geyer and Smith, 1987; Geyer and Farmer, 1989; Wesson
and Gregg, 1994; Seim and Gregg, 1994; Zhou, 1998]. The latter have proven to be a useful visualization tool of these instabilities.

As discussed by Geyer and Smith [1987], the difficulties associated with the observation of shear instabilities may be partially overcome by examining a natural system where a persistent shear is maintained and where the main external forcing is provided by the periodic action of tides, as in a stratified tidal estuary. Although complicated by unsteady flows and horizontal advection, such a system approaches controlled laboratory conditions under which experiments may be reproduced repeatedly under similar forcing conditions. If the basin is also hydraulically controlled by a sill or a constriction, shear instabilities may be generated by a variety of mechanisms. The interaction of the stratified fluid with abrupt topography perturbs the isopycnals and may be the source of a broad range of complex and interacting internal deformations such as lee waves, internal hydraulic jumps, density currents, high-frequency internal gravity waves, and internal tides. These topographically generated perturbations tend to destabilize the water column by increasing the vertical shear and thus constitute a source of energy for the development of shear instabilities. Many fjordlike systems that are hydraulically controlled by a sill or constriction at their mouths show such features [e.g., Farmer and Freeland, 1983]. One such basin [Saucier and Chassé, 2000] (herein referred to as SC), the St. Lawrence Estuary in eastern Canada (Figure 1), is examined in the present study.

The study of internal tides in the St. Lawrence Estuary has been the subject of continuous research since Forrester [1974] (see Mertz and Gratton [1990] for a review), while high-frequency internal gravity waves remain poorly documented. DeGuise [1977] first gave evidence of high-frequency internal wave activity near English Bank and, Ingram [1978] observed the sea surface signature of internal wave packets using aerial photographs off Ile Verte. More recently, Galbraith [1992] observed density inversions thought to be overturning events near the head of the Laurentian Channel.

As stated by Mertz and Gratton [1990, p. 97], “Work on internal waves in the St. Lawrence Estuary is still in its infancy. The complexity of the bottom topography in the upper estuary makes the interpretation of data from this region a formidable problem.” Complex topography can now be represented to increasingly fine scales by numerical models of the coastal environment, and these models may help
to orient field experiments and to interpret collected data. With increasing resolution it is possible to resolve the high-frequency internal gravity wave field down to wavelengths of a few hundred meters.

On the basis of a high-resolution, three-dimensional numerical model of the St. Lawrence Estuary, SC recently suggested the location and the timing of the occurrence of shear instabilities in the estuary. They demonstrated the existence of an intense and vertically sheared density undercurrent that develops semidiurnally in the north channel (see Figure 1). This current is produced every flood tide by the spilling of cold intermediate waters over the sill at the head of the Laurentian Channel. It accounts by itself for most of the tidal transport, with the surface layer remaining nearly still or slowly moving during flood. SC suggested that the vertical shear associated with this current was sufficient to destabilize the pycnocline, leading to Kelvin-Helmholtz instabilities.

In this model, mixing that results from the breaking of internal waves is a subgrid-scale process parameterized in terms of grid-scale variables. It follows that the presence of shear instabilities may only be inferred from the resolved scales. Low values of the gradient Richardson number give some indications of regions where these instabilities may be expected to occur. Thus the model does not resolve the scale of the overturning cells; uncertainties remain about the detailed structure of the high-frequency oscillations. Under these circumstances, field observations are needed to support the numerical results. The work of SC thus motivated our search for shear instabilities associated with the density current in the north channel of the St. Lawrence Estuary.

The objective of this paper is twofold: (1) to document, from fine-scale observations, the conditions under which shear instabilities can develop semidiurnally in the St. Lawrence Estuary and (2) to compare the results of a three-dimensional numerical model of the density, shear flow, stratification, and Richardson numbers with observations collected at the experimental site. An overview of the dynamics of the estuary and the region of interest is presented in section 2. Section 3 describes the experimental setup, and section 4 presents observations and numerical model results. A discussion follows in section 5, and the paper is concluded with section 6.

2. Baroclinic Circulation

The St. Lawrence Estuary is naturally divided into an upper and a lower basin separated by an abrupt sill (30 m) at the head of the deep (350 m) glacial valley called the Laurentian Channel (Figure 1). The upper estuary is partially stratified [Néel, 1970], and more than 90% of the variance of the currents is explained by tides alone [Muir, 1982; Chassé, 1994; Saucier et al., 1999; Saucier and Chassé, 2000]. In summer the lower estuary is typically stratified into three distinct layers: a brackish surface layer (0–30 m) due to freshwater discharge of the St. Lawrence and Saguenay Rivers, a cold intermediate layer (30–125 m) formed each winter through surface cooling in the Gulf of St. Lawrence [Banks, 1966; Gilbert and Pettigrew, 1997], and a warm and salty bottom layer that originates from the Atlantic Ocean [Lauzier and Trites, 1958; Koutonsky and Bugden, 1991]. The mean residual laterally averaged circulation of the whole estuary is estuarinelike, with the brackish surface layer flowing seaward and the cold intermediate layer flowing landward as well as the bottom layer, although at a smaller pace.

On tidal timescales a mixed barotropic tide propagates from the Atlantic Ocean into the estuary and interacts with

Figure 2. Model results showing the (top) Lagrangian currents and (bottom) surface density $\sigma_t$ during (left) ebb and (right) flood tide at the head of the Laurentian Channel and in the upper estuary.
Plate 1. Model results showing the spatial distribution of the vertically integrated turbulent buoyancy flux during (left) ebb and (right) flood tide.

Plate 2. Observed semidiurnal variability of the (a, b) axial $u$ and (c, d) transverse $v$ velocities (m s$^{-1}$), (e, f) density $\sigma_i$ (kg m$^{-3}$), and of the (g, h) density fluctuations $\sigma'_i$ (kg m$^{-3}$). Here $u$ and $v$ are positive in the seaward and southeastward direction, respectively (see Figure 1 for exact axes orientation). The abscissa on this and subsequent figures represents hours relative to high (hw) and low water (lw) at Pointe-au-Père. Thin, solid oblique lines in density panels mark drops and rises of the conductivity-temperature-depth probe. The roman numerals on top of this and subsequent figures refer to periods for which acoustic images are presented in text (section 4.3).
Plate 3. Simulated semidiurnal variability of the (a, b) axial \( u \) and (c, d) transverse \( v \) velocities (m s\(^{-1}\)), (e, f) density \( \sigma_t \) (kg m\(^{-3}\)), and (g, h) density fluctuations \( \sigma_t \) (kg m\(^{-3}\)).

Plate 4. Raw baroclinic velocity components (top) axial \( u' \), (middle) transverse \( v' \), and (bottom) vertical \( w' \), from 1144 to 1224 LT on August 20, corresponding to the same time window as the acoustic image of Figure 7g. Units are m s\(^{-1}\).
the shallow sill at the head of the Laurentian Channel. The interaction of the stratified tidal stream with the sill generates lee waves (SC), internal hydraulic jumps (SC), and internal tides [Forrester, 1974]. At flood tide $\approx 6.0 \times 10^8$ m$^3$ s$^{-1}$ of cold intermediate water is forced to flow over the sill (SC). Being gravitationally unstable relative to the surrounding fluid, this cold water flows down the landward slope and renews with cold water the inner basins of the north and south channels and of the Saguenay fjord.

Figure 2 and 3 illustrate from numerical results this tidal circulation. Ebb and flood surface currents and the surface density field are shown in Figure 2. The large asymmetry between ebb and flood surface currents is an indication of the strong baroclinic nature of the circulation. In the north channel, ebb surface currents reach intensities up to 2.0 m s$^{-1}$. The $V$-shaped convergence of the surface currents at flood tide, just east of Ile Rouge, marks where cold intermediate water brought close to the surface sinks into the north channel. At the same time a sharp, $V$-shaped surface density front (Figure 2), with 2 to 5 kg m$^{-3}$ density changes over a few meters, is formed and can be clearly identified from aerial photographs (SC). Other surface fronts were also reported around the Ile Rouge bank by Ingram [1976, 1985].

Figure 3 shows the ebb and flood vertical circulation and associated density field in the north channel along the cross section depicted by the solid line in Figure 1. The effect of the spilling of cold intermediate water and the formation of a bottom density undercurrent at flood tide are clearly identified on the figure. In this paper we show that this undercurrent provides sufficient vertical shear to destabilize the pycnocline.

Plate 1 shows the computed vertically integrated turbulent buoyancy fluxes

$$B_f = g \int_{-H}^{0} K_V \frac{\partial \sigma}{\partial z} dz,$$

where $g$ is the gravitational acceleration, $H$ is the total depth, $K_V$ is the vertical eddy diffusivity of density, $\sigma$ is the density, and $z$ is the vertical coordinate with its origin at the sea surface and being negative downward. During ebb tide, high levels of mixing are found on both the southeastern and northwestern side of Ile Rouge. The predicted high level of mixing there is attributed to the interaction of stratified and intense tidal currents with the topography and to localized sill processes (SC). During flood tide, high levels of mixing are found in the north channel southwest of Ile Rouge and in a region close to the southwestern end of Ile-aux- Lièvres (circle in Plate 1). SC suggested that the vertical shear at the pycnocline was responsible for the high level of turbulence there. Although their model does not resolve the scale of the instability, they suggested that mixing there is probably the result of breaking Kelvin-Helmholtz billows. In this paper we examine, in detail, the mechanism that leads to the predicted enhanced level of turbulence found near the southwestern end of Ile-aux-Lièvres.

3. Experimental Setup

3.1. Field Experiment

On August 19-20, 1997, an experiment was designed to measure the fine-scale, semi-diurnal variability of currents and density and to collect acoustic images at a site in the north channel close to the southwestern end of Ile-aux-Lièvres. The position is $47^\circ 50' 00"$N and $69^\circ 49' 00"$W, and the depth is 70 m. The position of the experiment is illustrated by the star in Figures 1 and 2 and by the vertical lines on Figure 3. For logistical reasons the experiment was split into two parts; we sampled the flood phase from 1250 to 1840 LT (here LT is the same as eastern standard time), on August 19 and the ebb phase from 0710 to 1400 LT on August 20. The experiment coincided with spring tide, and the data were acquired during calm wind conditions. Figure 4 shows the time spanned by the experiment. In the text and figures the time is expressed in hours relative to the tidal phase at Pointe-au-Père. High water (hw) is thus 1440 LT on August 19, and low water (lw) is 0920 LT on August 20, according to the Canadian tide and currents tables [Government of Canada, 1997b].
The three-dimensional currents are remotely measured with an RD Instruments 600-kHz range broadband acoustic Doppler current profiler (ADCP). The ADCP is deployed at 1.5-m depth on the side of the vessel. The ADCP sampled velocity vectors every second in 1-m layers from 3.5 to 62.5 m. The velocities are measured relative to the seafloor. The error on the raw velocity measurements is ±0.142 m s⁻¹ for the horizontal components and ±0.062 m s⁻¹ for the vertical one.

Density is computed from the measurements of a Sea-Bird conductivity-temperature-depth (CTD) probe. The CTD is dropped into the water column at a constant falling speed of ≈ 0.25 m s⁻¹, and the sampling frequency is set to 2 Hz. The error on the density measurements is ±0.06 kg m⁻³. In order to mimic the acquisition of velocity data, density measurements are averaged over the same depth intervals as the ADCP (over 1-m intervals, from 3.5 to 62.5 m).

As opposed to the ADCP, the CTD does not collect data simultaneously over the whole water column, common depths being revisited approximately every 5 min. Because the buoyancy period at the experimental site is of the order of 5 min, the density measurements do not resolve high-frequency internal waves. In order to obtain two comparable data sets, the velocity and density data are smoothed with a 10-min running average to filter out oscillations above the buoyancy frequency. Thus, in this study, fine scale refers to a vertical resolution of 1 m and a sampling interval of 10 min. The averaging process reduces the error on the velocity data to ±0.020 m s⁻¹ for the horizontal velocities and to ±0.010 m s⁻¹ for the vertical velocity.

Flow visualization is provided by a 120-kHz Biosonics echo sounder deployed on the other side of the vessel at 1.5 m under the surface. The echo sounder records the intensity of the backscattered signal that is reflected from suspended matter such as zooplankton, organisms, or particles larger than a few millimeters. Echo sounders have been used in the St. Lawrence Estuary for the detection of plankton and appear to be useful for the visualization of flow conditions and high-frequency internal gravity waves [Simard, 1985]. We refer to Medwin and Clay [1998] for details on acoustical methods.

Unfortunately, data are missing from 1500 to 1620 LT (hw+0.3 to hw+1.7) on August 19 because of power failure. The missing data for currents and density were obtained using linear interpolation for the sake of illustration.

### 3.2. Numerical Model

The numerical model used for the present study has been used by Saucier et al. [1999] to produce a tidal current atlas [Government of Canada, 1997a] and by SC to study
buoyancy effects in the St. Lawrence Estuary. The model was originally developed by Backhaus [1985], modified by Stronach et al. [1993], and improved and applied to the St. Lawrence Estuary by Saucier et al. [1999]. It is a level model at fixed heights with a free surface that uses finite differences to solve the primitive equations for the three-dimensional velocity \((u, v, w)\) and density \(\sigma_t\) fields. Vertical turbulent closure is achieved through the level 2 scheme of Mellor and Yamada [1982], and horizontal diffusivities are determined following Smagorinsky [1963]. The horizontal grid size is 400 m, and the vertical coordinate is discretized by 20 layers at depths of 5, 10, 15, 20, 25, 30, 35, 45, 55, 65, 75, 90, 105, 125, 150, 175, 200, 250, 300, and 350 m. The model domain extends from the upper limit of tidal influence, near Trois-Rivières, to Pointe-au-Père in the lower estuary. At the landward boundary we impose a constant freshwater discharge rate of \(1.14 \times 10^4 \text{ m}^3 \text{ s}^{-1}\). Along the seaward boundary the model is forced with tidal oscillations of the water level. The initial density field is interpolated from available climatological data. The model has been validated against 51 current meter records for velocities and density and against > 2000 surface drifters. The relative error for tidal currents is 15%. We refer to Saucier et al. [1999] and SC for a detailed description of the model implementation, calibration, and validation.

We ran the model for the month of August 1997 and sampled the solution over the location and time period of the experiment (August 19-20, 1997). The solution was postaveraged from the model time step of 1 min to the 10-min intervals corresponding with the observations. The numerical results presented here were obtained using the same parameterizations as SC, and we did not perform any posttuning to better fit the present observations.

4. Comparison of Results From Observations and Numerical Model

4.1. Currents and Density

Plate 2 shows the observed semidiurnal variability of the axial \(u\) and transverse \(v\) velocities components (see Figure 1 for axes orientation), of the density \(\sigma_t\), and its fluctuations \(\sigma'_t\):

\[
\sigma'_t = \sigma_t - \bar{\sigma}_t,
\]

where \(\bar{\sigma}_t\) is the averaged density profile over the tidal cycle. Because of the periodic action of tides, the right and left panels on this and subsequent figures can be viewed as one continuous contour plot, even though there is a gap of 12 hours and 30 min separating them. Plate 3 shows the corresponding numerical results.

Qualitatively, we see that the model reproduces well the observed variability in time and through the water column of the axial and transverse currents (Plates 2a-2d and 3a-3d). The pattern of the evolution of the density is also qualitatively well reproduced. However, the density fluctuations are underestimated by a factor of \(\approx 2\) (Plates 2g-2h and 3g-3h).

The currents are mainly oriented along the axial axes with \(|u|\) reaching up to 2 m s\(^{-1}\) during ebb flow, while transverse currents are relatively small with \(|v| < 0.25 \text{ m s}^{-1}\). From hw-1 to hw+1 an intense upstream underrcurent, below 30 m, is observed and modeled. This underrcurent is produced by the spilling of dense water at flood tide over the sill, 35 km seaward (see Figure 2 for a broader perspective). The core of this underrcurent is located at around 40 m and reaches a maximum magnitude of -1.6 m s\(^{-1}\) at hw, while the surface layer is nearly motionless. During the period of neap tide, the surface currents (0-5 m) do not even reverse in this region, according to the atlas of tidal currents [see Government of Canada, 1997a, pp. 5-8].

The resulting forces due to the sum of the barotropic and opposing baroclinic pressure gradient cause the surface layer (0-30 m) to first reverse between hw+2 and hw+3, while the bottom layer still flows landward (Plate 2a). The intensity and the timing of this sudden change in flow condition is also well depicted by the model (Plate 3a). As the flow evolves, the barotropic pressure gradient dominates over the mean baroclinic component, and the whole water column flows seaward at speed \(\approx 2.0 \text{ m s}^{-1}\), from lw-2 to lw+2.

Around hw+3 the baroclinic pressure gradient due to the spilling of dense water over the sill 35 km to the northeast causes the velocity in the bottom layer, below 30 m, to reverse sign (Plate 2b). Again, the timing of this reversal and the intensity of the simulated currents (Plate 3b) are in good agreement with the observations.

4.2. Shear, Stratification and Gradient Richardson Number

We now consider the vertical shear squared,

\[
S^2 = \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2,
\]

and the buoyancy frequency squared,

\[
N^2 = -\frac{g}{\rho} \frac{\partial \rho}{\partial z},
\]

where \(z\) is the vertical coordinate taken positive in the upward direction and \(g\) is the gravitational acceleration. This definition of \(S^2\) in (2) is taken from Peters [1999] and the overbar denotes 10-min averaging. The vertical derivative in (3) was computed over Thorne-sorted profiles to remove overturns in order to assure real values of the buoyancy frequency [Thorpe, 1977].

Figures 5a-5d and 6a-6d show the observed and computed shear and stratification, respectively. The maximum observed shear \(S^2_o = 10 \times 10^{-3} \text{ s}^{-2}\) is found at middepth between hw-1 and hw and coincides with the maximum observed stratification with \(N^2_o = 4.0 \times 10^{-3} \text{ s}^{-2}\). The corresponding maximum modeled values at middepth during flood are \(S^2_m = 4.5 \times 10^{-3} \text{ s}^{-2}\) and \(N^2_m = 1.5 \times 10^{-3} \text{ s}^{-2}\). Thus the model underestimates by approximately a factor of 2 the intensity of \(S^2_o\) and \(N^2_o\). This is not surprising, since the
observed velocities and density vary significantly over only a few meters, and the model’s vertical resolution around the pycnocline is only 5 m.

As the flow evolves to hw+2, the shear decreases and the stratification shows a double-layer structure. A first extremum with \( N^2_0 = 1.5 \times 10^{-3} \text{ s}^{-2} \) is observed around 25 m, and a second one with \( N^2_0 = 1.75 \times 10^{-3} \text{ s}^{-2} \) is observed around 40 m (Figure 5c). Although there may be many reasons for multiple extrema in \( N^2 \), this double-layer stratification might be an indication of breaking Kelvin-Helmholtz billows; this has been observed in the atmosphere [Browning and Watkins, 1970] and in numerical experiments [Sykes and Lewellen, 1982]. The numerical model does not reproduce this fine-scale double structure of the pycnocline but, rather, diffuses the pycnocline over this time interval.

In the model results the passage of a thin layer of fresher water, from hw-2 to hw+2, creates highly stratified conditions near the surface with \( N^2_m = 1.5 \times 10^{-3} \text{ s}^{-2} \), which is not observed. We recall that observations from hw+0.3 to hw+1.7 were linearly interpolated and are subject to uncertainties. More observations and further analysis would be needed to determine whether this layer produced by the model is realistic and to determine its origin. Also, from hw+1 to hw+2 the model produces large shear near the surface (\( S^2_m \approx 3.0 \times 10^{-3} \text{ s}^{-2} \)) that is not observed. Again, this period corresponds to the period for which the observations were interpolated.

From lw-2 to lw+2 the modeled and observed vertical shear are relatively weak with \( S^2_m \approx S^2_0 \approx 1.0 \times 10^{-3} \text{ s}^{-2} \), and the maximum observed stratification is found near the surface with values reaching \( N^2_0 = 3.0 \times 10^{-3} \text{ s}^{-2} \). The model underestimates by a factor of 2 this near-surface stratification with \( N^2_m = 1.5 \times 10^{-3} \text{ s}^{-2} \).

At the reversal of the barotropic pressure gradient at lw+3, the column restratifies and the shear increases owing to the renewal of cold intermediate water. This change in \( S^2 \) and \( N^2 \) is well depicted by the model but underestimated.

Although the magnitudes of the model shear and buoyancy frequency are underestimated, the stability of the flow is determined by the ratio of the square of these two quantities. This ratio defines the gradient Richardson number \( Ri = N^2 S^{-2} \). Using linear theory, Miles [1961] showed that for two-dimensional, stably stratified, laminar, parallel shear flow, stability results when \( Ri > 0.25 \). However, the stability of a natural flow should not only depend on \( Ri \) but also on the Reynolds and the Péclet numbers [Sullivan and
List, 1993] and on internal wave activity. These nonlinear effects tend to destabilize even the water column, and naturally occurring shear instabilities are likely to develop when \(0.25 \leq Ri \leq 0.5\), but the exact value depends on the flow itself. We refer to Geyer and Smith [1987] for a historical perspective on the stability of naturally occurring stratified shear flow. Furthermore, the \(Ri\) value at which a stratified shear flow becomes unstable is dependent on the scale at which it is measured. Geyer and Smith [1987] showed, using fine-scale measurements in the Fraser Estuary, that if internal wave fluctuations are resolved in the evaluation of \(Ri\), then the threshold value should be close to 0.25. However, when \(Ri\) is evaluated based on mean conditions (i.e., over several buoyancy periods), the threshold value increases to 0.33. Because our measurements do not capture internal wave fluctuations, we consider the range \(0.25 \leq Ri \leq 0.5\) to be representative of instability.

Figures 5e-5f, and 6e-6f, show the observed and computed Richardson number, respectively. The shaded areas depict regions where \(0.25 \leq Ri \leq 0.5\). Considering the errors on the velocity and density measurements (see section 3), we evaluated that the error in \(Ri\) is \(\pm 0.06\).

During the flood of relatively dense waters the observed pycnocline is unstable between hw-1 and hw+3.5, which suggests that shear instabilities are likely to develop in this time interval. Uncertainties remain though about the stability of the water column for the period for which the observations were interpolated, from hw+0.3 to hw+1.7. Nevertheless, it is reasonable to think that the column was also unstable in that time period. The numerical results for \(Ri\) (Figure 6e and 6f) further support this hypothesis. From lw-2 to lw+2 the water column is stable in stratified regions with \(1.00 < Ri < 2.0\), and little mixing across the pycnocline is expected to occur. Values below \(Ri < 0.5\) are observed only in weakly stratified regions. At lw+3 the pycnocline suddenly becomes unstable again, and the cycle is completed. The acoustic images presented in the following section confirm some of these findings. The modeled pattern and \(Ri\) values are comparable with the observations (Figure 6e and 6f).

4.3. Flow Visualization

Figures 7a-7g show seven acoustic images at the time identified by the roman numerals (I-VII) on top of Plates 2
Figure 7a. Acoustic image I, from 1247 to 1327 LT on August 19, showing growing Kelvin-Helmholtz instabilities at an early stage. The oblique lines are reflections from the CTD probe. The wave train is centered around the pycnocline at 30 m, where the gradient Richardson number lies in the interval $0.25 \leq Ri \leq 0.5$. The wavelength is estimated as $\lambda \approx 138$ m.

and 3 and Figures 5 and 6. Figure 7a shows a sinusoidal wave train at around 25 m that exhibits characteristics of an early stage of amplifying Kelvin-Helmholtz instabilities. At this time the observed gradient Richardson number at the pycnocline just starts to be in the range where shear instabilities are expected to occur. If these waves are due to shear instabilities, laboratory and numerical experiments show their wavelength is $\lambda \approx 2\pi h$ [Turner, 1973], where $h$ is the thickness of the shear layer. We see from Figure 8 that $h = 22$ m, and the wavelength is thus evaluated around $\lambda \approx 138$ m.

Figure 7b. Acoustic image II, from 1328 to 1407 LT on August 19, depicting the turbulent mixing layer just after acoustic image I of Figure 7a.
Figure 7c. Acoustic image III, from 1435 to 1501 LT on August 19, depicting large Kelvin-Helmholtz billows centered at the pycnocline. The wavelength between successive billows is evaluated to be around \( \lambda \approx 161 \text{ m} \).

Figure 7b shows the evolution of the following 39 min. At this time the wave train of Figure 7a has collapsed to a mixing layer at \( \approx 25 \text{ m} \). The mixing layer has a vertical extent of \( \approx 10 \text{ m} \). The Richardson number of the pycnocline is now well within the interval \( 0.25 \leq Ri \leq 0.5 \). Braid structures associated with the breaking of Kelvin-Helmholtz billows, similar to the one observed by Geyer and Smith [1987] and Geyer and Farmer [1989], are also identified on the figure. Figure 7c represents a 25-min image one-half hour after Figure 7b. At least four Kelvin-Helmholtz billows are identified, similar to the one observed by Seim and Gregg [1994]. At this time both the shear and stratification have decreased.

Figure 7d. Acoustic image IV, from 1719 to 1751 LT on August 19, showing mixing activity and Kelvin-Helmholtz billows.
slightly but the Richardson number is still low at value of 0.30 < Ri < 0.35 (see Figure 5). By this time the shear layer thickness has increased slightly to 24 m (Figure 8) and the wavelength is estimated to be around $\lambda \approx 151$ m. Figure 7d shows again clear evidence of Kelvin-Helmholtz billows and mixing activity at the reversal of the barotropic pressure gradient, again associated with low $Ri$ values at the pycnocline.

As discussed in section 4.2, shear instabilities are not expected to occur between lw-2 and lw+2. This is consistent with Figures 7e and 7f, which show quiet conditions during the ebb flow. However, a traveling wave train is observed toward the left of Figure 7e. The shape of this wave packet indicates it has been generated by tidal flow over topography, but its exact origin is unknown. Such wave packets are common in the coastal environment [e.g., Haury et al., 1979; Farmer and Smith, 1980; Farmer and Freeland, 1983; Stacey and Zedel, 1986].

Figure 7g shows a large wave train during the transition from ebb to flood tide between lw+2 and lw+3. The vertical displacement reaches nearly 50 m, more than 2/3 of the total depth, indicating nonlinearity. The interaction of the wave with the vertical shear gives rise to shear instabilities observed toward the right of the figure. At this time the observed and simulated gradient Richardson numbers become low again (Figures 5f and 6f).

Figure 7f. Acoustic image VI, from 1056 to 1117 LT on August 20, depicting quiet conditions just before the reversal of the tidal stream.
Plate 4 shows the observed raw baroclinic velocity
\[(u', v', w') = (u, v, w) - (\bar{u}, \bar{v}, 0)\]
associated with this wave. The overbars denote depth averages. As noted in section 3, the error on the raw velocity component is \(\pm 0.142 \text{ m s}^{-1}\) for the horizontal components and \(\pm 0.062 \text{ m s}^{-1}\) for the vertical one. Crests and troughs are associated with negative and positive vertical velocity, respectively, with maximum amplitudes equal to 0.1 \text{ m s}^{-1}. The horizontal baroclinic velocity components \(u'\) and \(v'\) oscillate in phase, with nearly equal amplitudes reaching 0.4 \text{ m s}^{-1}. Thus the main horizontal axis of oscillation is approximately at 45° to the right of the axial axis. This is about the same orientation as the maximum local gradient of the topography (see Figure 1). A hypothesis for the origin of this large wave train is discussed below.

5. Discussion

Numerical results suggest that the spilling of cold intermediate water during flood tide over the sill at the head of the Laurentian Channel creates an intense gravity current in the north channel. The numerical results also suggest that the vertical shear associated with this strong tidal, baroclinic circulation was sufficient to destabilize the pycnocline and enhanced mixing. It was suspected that shear instabilities provided the mixing mechanism.

Guided by the numerical results, a field experiment was undertaken to measure the fine-scale vertical and temporal variabilities at one site in the north channel where instabilities were expected to occur. The instabilities have been observed at the predicted location and time, as evidenced from acoustic imagery. The instabilities have a wavelength between 140 and 150 m and extend vertically between 10 and 25 m. The model results for currents, density, and gradient Richardson numbers appear to be in good agreement with the observations. The model underestimates the shear and the stratification by a factor of \(\approx 2\).

We did not make any attempt to compare the simulated and observed turbulent fluxes of momentum and mass, lacking any mean to evaluate accurately the dissipation rate of turbulent kinetic energy with the present data. Future experiments must incorporate microstructure-measuring devices [e.g., Seim and Gregg, 1994; Peters, 1997, 1999; Trowbridge et al., 1999; Lu et al., 2000; Peters and Bokhorst, 2000] to achieve this goal. Furthermore, the turbulent boundary layer model used in our model (level 2) [Mellor and Yamada, 1974, 1982] is not appropriate for low Reynolds number turbulence. Inclusion of a model for shear-induced mixing, such as the improved model of Kantha and Clayson [1994], should be considered before attempting to compare turbulent quantities.

A new and unexpected finding is the existence of large internal wave trains propagating in the north channel at the transition from ebb to flood tide (see Figure 7g and Plate 4). The wave train we have observed looks as if it was induced by a tidally forced flow over a sill or a bank [e.g., Farmer and Freeland, 1983]. The amplitude of this wave is such that it cannot have been generated at the sill at the head of the Laurentian Channel, 35 km away. One possible mechanism is the interaction of the stratified tidal flow with the local topography. This process of internal wave generation was demonstrated by Lamb [1994] from numerical experiments. We use his results to interpret our observations.

Figure 9 shows the local along-channel topography around the experimental site. The bank seen in the middle of the figure is similar in shape to Lamb's [1994] setup, with depth changes of \(\approx 50\) m over 5 km. The existence of large
internal waves in this region could be explained by a sudden depression of isopycnals that forms over the bank edge during off-bank flow (i.e., during flood tide in our case). The depression then propagates against the flow, in the seaward direction, where it becomes supercritical on top of the bank. The depression gets trapped at the bank edge, the amplitude increases, and the wavelength decreases [Lamb, 1994]. Supercritical conditions are found when the Froude number $Fr \equiv Uc^{-1} > 1$, where $U$ is the vertically averaged flow speed and $c$ is the phase speed of the long internal wave. For a two-layer system the phase speed can be approximated by [e.g., Porter and Thompson, 1999],

$$c \approx \sqrt{\frac{g}{p2} \frac{h_1 h_2}{h_1 + h_2}}. \quad (4)$$

Just before the arrival of the wave train, at $lw+2$, the upper and lower layer thickness are $h_1 \approx 10$ m and $h_2 \approx 60$ m, and the corresponding densities are $p_1 \approx 1020.0 \text{ kg m}^{-3}$ and $p_2 \approx 1022.0 \text{ kg m}^{-3}$ (see Plate 2f). According to (4), this gives a phase speed of $c \approx 0.4 \text{ m s}^{-1}$. Thus, with $U \approx 0.5 \text{ m s}^{-1}$, the flow is supercritical ($Fr \approx 1.25$). These observations are in accordance with Lamb’s [1994] theoretical model and support the idea of waves being generated locally. The generation process over the bank is illustrated schematically in Figure 9. This model provides a plausible mechanism for the generation of such waves; however, because of the complexity of the topography in the upper estuary, there are many other places around where such a wave could have been generated.

This finding of the existence of large, nonlinear internal wave trains provides an example illustrating the limitations of numerical models in reproducing the fine-scale structure of the high-frequency internal gravity wave field of the coastal environment. With a horizontal resolution of 400 m, the numerical model used in this study does not reproduce the details of the fine-scale features observed in the St. Lawrence Estuary. There are mainly two reasons for these modeling difficulties. First, the model would need a horizontal resolution of the order of 50 m in order to resolve the internal structures that exhibit typical spatial scales of $\sim 100 - 200$ m. Second, the hydrostatic approximation is violated because the frequency of the wave train seen in Figure 7g ($\omega \approx 0.02 \text{ s}^{-1}$) is not small compared with the buoyancy frequency ($N \approx 0.04 \text{ s}^{-1}$) [see Gill, 1982, p. 259]. Under this condition the hydrostatic approximation is no longer valid, and, at these scales, one should consider the use of a nonhydrostatic model.

6. Conclusion

In this paper we have demonstrated the ability of a numerical estuarine model to reproduce the correct ratio of fine-scale shear and stratification, even though the model resol-
tion is too coarse to resolve the detailed high-frequency wave field. In other words, the mean gradient Richardson number, which determines the threshold for shear instabilities to develop, is well reproduced. From a numerical point of view this is an important result, because correlations are expected between the dissipation rate of turbulent kinetic energy $\epsilon$ and $\bar{R}i$ for shear instabilities [Miles, 1961; Gregg, 1989; Peters, 1997]. Mixing that results from such instabilities can then be parameterized by relating $\epsilon$ or $\bar{R}i$ to the coefficients of turbulent viscosity and diffusivity [Osburn, 1980; Munk and Anderson, 1948; Pacanowski and Philander, 1981]. Although the exact dependance is subject to large uncertainties, this approach is still the best that can be done currently to treat shear-induced mixing in numerical circulation models [e.g., Kantha and Clayson, 1994; Large and Gent, 1999]. One step has been achieved here, but clearly, more fine-scale and microscale measurements taken under variable forcing conditions (high-low river flow, neap-spring tidal cycle) are needed to calibrate these models.

For the first time the conditions that favor pycnocline mixing in the upper St. Lawrence Estuary have been described. As predicted by the numerical model, we find evidence that mixing indeed occurs within the shear layer produced by a bottom density current made of invading dense waters over the sill during flood.

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References


Muir, L. R., Internal tides in a partially mixed estuary, *Tech. Rep.* 9,
Stacey, M. W., and L. J. Zedel, The time-dependant hydraulic

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